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Endogenous growth and education financing

(first draft)

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1. Introduction

What are the factors that determine a country's rate of economic growth in the lung run? The growth model's in the 1960s assigned a significant role to private investment in physical capital accumulation. Lung-run growth in these models was entirely due to growth in technological progress, which was exogenous to the models. By contrast, contemporary growth theory assigns an important role to accumulation of human (knowledge) capital, at both the aggregate and the individual levels. Moreover, models of 1980s and 1990s generate lung run growth from the actions of individuals in the economy (see Romer 1986; Lucas 1988).

In this theoretical context economists have begun to study the influence of education spending on consumption-saving decisions in models which allow the possibility of persistent growth (Glomm and Ravikumar 1992). The recent developments have significant policy implications since public or private expenditures on education may influence lung-run growth and social welfare. To the extent that formal schooling is a significant component of human capital investment, the institutions for schooling may be important for growth (Grandstein, Justmann and

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Mayer 2005). The aim of this paper is that to analyze in a simple context the relevance of education financing to make it possible to model endogenous growth, even when physical capital accumulation encounters decreasing returns to scale.

2. The general framework

The motivation for modeling human capital (education) in an aggregating setting is that the lung-run growth of the economy may be endogenously explained, even when the returns to physical investment alone are decreasing. Consider the standard neoclassical aggregate production function relating aggregate output to aggregate stock of physical capital and economy's workforce, $Y = F(K, AL)$, where A indexes labor-augmenting technological progress.

Neoclassical growth models assume that the function $F(K, L)$ has constant returns to scale in its two arguments, so that it is feasible to compensate private factors of production to their true marginal productivities. But also we have that

$$\lim_{K \to \infty} F'(K) = 0 \qquad [1]$$

i.e., the marginal productivity of the accumulate factor decreases to zero as the economy's stock of capital grows. Then, per capita income grows like A and, to model sustained growth, the labor-productivity index must increase over time so that new capital goods offsets the tendency of marginal productivity of physical capital to decrease.

The endogenous growth perspective leads to abandonment to the assumption of overall constant returns to scale and assigns an important role to human capital accumulation. Building on earlier work by Arrow (1962) and Uzawa (1965), Lucas (1988) and Rebelo (1991) replace the effective labor force variable AL by a human capital aggregate. The production function become the following:

$$Y = F(aK, bH) \qquad [2]$$

where aK and bH respectively represent physical and human capital devoted to the final output. Importantly, H is neither an exogenous provided good nor a pure public good. Economic theory can endogenously explain lung run growth if it can model the provision human capital (i.e. education).

Lucas (1988) takes the extreme case where the only input in human capital production is human capital itself, so the accumulation equation of reproducible factor, *at the aggregate level*, assumes the linear form:

$$\dot{H} = \delta(1-b)H \qquad [3]$$

where δ is a productivity parameter; we can observe that in this case the rate of return to both human and physical investment is pinned down by the productivity δ of the existing human capital in the production of new human capital. In this case the linearity assumption guaranties that economy's growth rate is unrelated to the parameter of production function. In these models, the speed of growth need not to be inversely related to physical capital intensity. In essence, the human capital factor plays the some role of exogenous technological progress in earlier models, namely that of offsetting physical capital's decreasing marginal productivity.

3. The macroeconomic role of education in a simple endogenous growth model with homogeneous agents

In this paragraph we begin with a two-period model of private education as investment in human capital that will serve to point out the relationship between endogenous growth and education spending. Consider an economy populated with a unit measure of households, each consisting of a parent and a child, in which all decisions are made by parents who determine the amount of investment in their children's schooling. Parents invest in their children's schooling from a bequest motive, deriving utility from consumption c_t, from the consumption in the next period c_{t+1} and from their offspring's education investment E. Assigning the same utility function to all parents, we write

$$ln(c_t) + log(E) + \beta ln(c_{t+1}) \qquad [4]$$

This formulation abstracts from important issues regarding the relationship between school spending and the quality of schooling and from differences in children's innate abilities. Parents then allocate their income between consumption, present and future, and education reflecting the assumption that the credit constraints do not allow them to borrow against their children's future income, so that private education is exclusively financed by parental income. Normalizing all prices to unity, we have the budget constraints

$$c_t + s_t + E = w_t h_t$$

$$c_{t+1} \leq (1 + r_{t+1}) s_t \qquad [5]$$

where s_t is the individual saving and each individual takes the wage rate, w, as given.

Members of each generation are endowed with one unit of leisure in their youth and h_t units of human capital. Human capital is accumulated according to the production function

$$h_{t+1} = H(h, E) = h_t^{\gamma} E^{1-\gamma} \qquad [6]$$

where E is public expenditure on education. The key element is that in equation [6] human capital stock can be augmented throughout the resources allocated to education. Each firm produces output y at time t according to the standard technology,

$$y = Ak^{\alpha}(nh)^{1-\alpha} \qquad [7]$$

where k is the amount of capital rented by the firm, nh is the amount of skill-weighted or effective labor input. Under private schooling, utility maximization subject to intertemporal constraint yields the following expression

$$s = \frac{\beta}{2+\beta} w_t h_t \qquad [8]$$

Profit maximizations by firms yields

$$w_t h_t = (1-\alpha) A k_t^{\alpha} h_t^{1-\alpha} \qquad [9]$$

In equilibrium $k_{t+1} = s_t$,

$$k_{t+1} = \frac{\beta}{2+\beta}(1-\alpha) A k^{\alpha} h^{1-\alpha} = \Theta k^{\alpha} h^{1-\alpha} \qquad [10]$$

The optimal amount of private education spending by parents is

$$E = \frac{1}{2+\beta}(1-\alpha) A k^{\alpha} h^{1-\alpha} \qquad [11]$$

Inserting this last equation [11] in the equation expressing the accumulation of human capital [6] we obtain

$$h_{t+1} = \left(\frac{1-\alpha}{2+\beta}\right)^{1-\gamma} A^{1-\gamma} h^{\gamma} (k^{\alpha} h^{1-\alpha})^{1-\gamma} = \Omega h^{\gamma} (k^{\alpha} h^{1-\alpha})^{1-\gamma} \qquad [12]$$

Thus, the ratio of human capital to physical capital evolves according to

$$\frac{h_{t+1}}{k_{t+1}} = \frac{\Omega}{\Theta} \frac{h^{1-\alpha+\alpha\gamma} k^{\alpha(1-\gamma)}}{k^{\alpha} h^{1-\alpha}} = \frac{\Omega}{\Theta}\left(\frac{h}{k}\right)^{\alpha\gamma} \qquad [13]$$

Clearly, this ratio converges monotonically to a unique steady state, $x^*_{priv} = \frac{\bar{h}}{\bar{k}}$, given by

$$log\, x^* = \frac{1}{1-\alpha\gamma} log \frac{\Omega}{\Theta} \qquad [14]$$

Consumption, physical capital, human capital and output all grow at the same rate. Substituting in the equation [10] the lung run growth rate is given by

$$g_{priv.} = log\,\Theta + (1-\alpha) log\, x^*_{priv} \qquad [15]$$

The economy does not converge to a steady state level but to a sustained growth. In this case the decreasing returns of capital are compensated by the contrasting effects of human capital. The model predictions are similar to the endogenous growth literature.

4. The case of public education

In this paragraph we extend the framework described in the previous section for the case of public schooling. We focus on the pure public education, excluding all private acquisition of education, assuming that public schooling is financed by a proportional income tax determined by majority voting among parents, where t denotes the tax rate and $0 \le t \le 1$. Every individual born at time t has identical preferences represented by

$$ln(c_t) + \beta\, ln(c_{t+1}) \qquad [16]$$

where c_i is consumption of an individual at time i.

The constraints faced by a representative young agent are

$$c_t + s_t = (1-t)w_t h_t \qquad [17]$$

$$c_{t+1} \leq (1+(1-t)r_{t+1})s_t \qquad [18]$$

where each individual takes the wage rate, the real interest rate, and the tax rate as given. Maximizing utility [16], subject to the budget constraints [17] and [18], we obtain the household's optimal amount of saving

$$s_t = \frac{\beta}{1+\beta}(1-t)wh \qquad [19]$$

In equilibrium $k_{t+1} = s_t$

$$k_{t+1} = \frac{\beta}{1+\beta}(1-t)(1-\alpha)Ak^{\alpha}h^{1-\alpha} \qquad [20]$$

Under the public system, a government levies taxes on a national wide basis and uses revenues to finance education spending. All children receive the same amount, which is collected by levying a proportional tax on income,

$$E = tAk^{\alpha}h^{1-\alpha} \qquad [21]$$

and hence, considering equation [6],

$$h_{t+1} = h^{\gamma}(tAk^{\alpha}h^{1-\alpha})^{1-\gamma} \qquad [22]$$

Thus, the ratio of human capital to physical capital evolves according to

$$\frac{h_{t+1}}{k_{t+1}} = \frac{(1+\beta)t^{1-\gamma}}{\beta(1-t)(1-\alpha)} A^{-\gamma}\left(\frac{h_t}{k_t}\right)^{\alpha\gamma} = \Lambda\left(\frac{h_t}{k_t}\right)^{\alpha\gamma} \qquad [23]$$

As in the previous case this ratio converges monotonically to a unique steady state, $x^*_{pub} = \frac{\bar{h}}{\bar{k}}$, given by

$$log\ x_{priv}{}^* = \frac{1}{1-\alpha\gamma} log\ \Lambda \qquad [24]$$

Consumption, physical capital, human capital and output all grow at the same rate. Substituting in the equation [20] the lung run growth rate $g_{pub.}$ is given by

$$g_{pub.} = log\frac{\beta}{1-\beta}(1-\alpha)A + log(1-t) + (1-\alpha)log\ x^*_{pub} \qquad [25]$$

As in the private scheme, the economy does not converge to a steady state level but to a sustained growth. The dynamics in this economy looks similar to the previous model with private expenditure on education. The reason is the same: the accumulation of human capital compensates the decreasing returns of physical capital. The education financing, both private or public, is a social mechanism able to sustain steady state growth in the long period.

5. A comparison between the two systems of education financing

In this paragraph we compare the equilibrium path income for the two educational systems. We have to distinguish two cases. In the first case the initial generation is homogeneous, that is the initial distribution of income is degenerate so that per capita income coincides with the representative agent's income. The purpose is to abstract from distributional issues and compare the levels and growth rates of income in the two educational systems. In the second case, we consider the heterogeneous distribution and analyze the evolution of inequality over time.

With respect the first issue, the comparison between the two financing schemes is made easy considering the fact that the equations representing the growth rates have the same structure. From equations [15] and [25] the evolution of the ratio of human capital to physical capital in both economies is similar to the capital accumulation equation in the Cass-Koopmans model. From a direct comparison we are able to conclude that the private education financing produces a higher growth rate if

$$log\,\Theta + (1-\alpha)\,log\,x^{*}_{priv} > log(\frac{\beta}{1-\beta}) + log(1-t) + (1-\alpha)\,log\,x^{*}_{pub} \qquad [26]$$

In the last equation the left term is constant, while the right term is depending upon the tax rate chosen by the representative agent. In contrast with the case considered by Glomm and Ravikumar (1992), in which the private education economy did have always a higher growth rate, in this model this effect is depending on the level of taxation, t.

Probably the main differences between the two approaches spring from the distributional implications in the long period. Public education provides an uniform level of schooling, thus it reduces inequality within cohorts and increases intergenerational mobility. On the contrary, in the private education scheme, as income shares do not change over time, income inequality remains constant (in the sense that the Lorenz curve is constant). In order to consider the dynamic aspects of the private scheme we have to substitute equations [9] and [11] in the human capital equation [6]. The result is the following expression

$$h_{t+1} = Bh^{\gamma}(\frac{1}{2+\beta}wh)^{1-\gamma} = B\left(\frac{1}{2+\beta}\right)^{1-\gamma} w^{1-\gamma}h \qquad [27]$$

If the human capital at time t is log normally distributed with mean μ and variance $\sigma_t^{\ 2}$, then the human capital at time $t+1$ is also log normally distributed with mean μ_{t+1} and variance $\sigma_{t+1}^{\ 2}$ where

$$\mu_{t+1} = \mu_t + \log B + (1-\gamma)\log(\frac{1}{2+\beta}) + (1-\gamma)\log w \qquad [28]$$

and

$$\sigma_{t+1}^2 = \sigma_t^2 \qquad [29]$$

The last equation shows that when the resources devoted to the education only rely on the child's family income, there are no convergence forces at work; inequalities as measured by the variance of the logarithmic of human capital remains constant over time. As the child's income is perfectly correlated with the parent's income, we can say that there is no intergenerational mobility in this case.

Quite different are the distributional effects in the public financing scheme. First of all, we assume that the tax rate is constant and determined by means of majority voting. As the resulting tax rate does not depend on the type of household ant it is constant. Each household thus benefits from the following amount of public education

$$\overline{E} = \frac{t}{1-\alpha} w\overline{h} \qquad [30]$$

Proceeding as in the previous case, individual human capital will accumulate according to

$$h_{t+1} = Bh^{\gamma}(t\frac{w\overline{h}}{1-\alpha})^{1-\gamma} = B\left(\frac{t}{1-\alpha}\right)^{1-\gamma} h^{\gamma} w^{1-\gamma} \overline{h}^{1-\gamma} \qquad [31]$$

and the evolution of the parameter in this case is the following

$$\mu_{t+1} = \gamma\mu_t + \log B + (1-\gamma)\log(\frac{t}{1-\alpha}) + (1-\gamma)\log w + (1-\gamma)\log \bar{h} \qquad [32]$$

and

$$\sigma_{t+1}^2 = \gamma^2 \sigma_t^2 \qquad [33]$$

The last equation help as to characterize the evolution of income inequality over time. In the model, income inequality at every point of time is described by the parameter $\sigma_t{}^2$. In the public education economy income inequality declines over time because $\gamma < 1$, by assumption. The public system has a redistributive property, providing a uniform level of schooling. Public education is an instrument of upward mobility for children to poorer backgrounds and a force working to reduce income inequality.

6. The poverty trap

Distribution also does matter when access to financial markets is prohibited and investment must be self-financed. In that case it may be that, for some households, saving is not enough to undertaken the high-return human capital investment. The financial markets relevant to human-capital accumulation can be imperfect in two different respects. First, an agent cannot borrow an unlimited amount of capital at the interest rate which he receives when he is a lender rather than a borrower: he is constrained in the amount of capital, or the interest rate on borrowed capital is larger than on loan. Secondly, when the outcome of a given investment in education is random, the risk associated with the investment is non-insurable. When the financial market imperfections make it impossible to reap the fruits of investment on child's education, the economy's resources are non allocated efficiently. The required investment in education is to high and the poor individuals can not afford education. The final result can be the poverty trap, a steady state feature with an unequal distribution of wealth across households and a *class society* (Bertola, Foellmi and Zweimüller, 2006).

The previous model offers a simple way to consider also this more realistic case. Assume that investment in education is ineffective below some minimal threshold level of human capital h^*. So, for that part of population with $h > h^*$, the accumulation equation of human capital remains the [12]. If the human capital is below the critical level, $h < h^*$, saving fall short the amount required to undertake the high-return education good. In this case we assume $E = 0$ and the equation of human capital becomes the static one, $h_{t+1} = h_t = h^*$. Everyone would be educated but some households are too poor to undertake the indivisible investment.

The poor household solves the problem di maximize the following utility function

$$ln(c_t) + \beta\, ln(c_{t+1}) \qquad [34]$$

under the usual budget constraint

$$c_t + s_t = w_t h_t$$

$$c_{t+1} \leq (1 + r_{t+1}) s_t \qquad [35]$$

In equilibrium $k_{t+1} = s_t$,

$$k_{t+1} = \frac{1}{1+\beta} wh = \frac{1}{1+\beta} (1-\alpha) k^{\alpha} h^{*1-\alpha} \qquad [36]$$

From this equation we can derive the economy growth rate

$$\frac{k_{t+1}}{k_t} = \frac{1}{1+\beta}(1-\alpha)\frac{h^{*1-\alpha}}{k^{1-\alpha}} \quad [37]$$

In the lung run

$$lim_{k\to\infty}\frac{k_{t+1}}{k_t} = 0 \quad [38]$$

because the decreasing returns of capital, the only accumulated factor. In this model the dynamics of educational investment is simple: individuals that, initially, own sufficient human capital to educate their children will converge toward the steady state growth, whereas households who are too poor will converge toward a zero growth steady-state. The growth rate of the economy will be a mean of the growth rates of the two groups.

7. **Concluding remarks**

In this contribution we have presented a simple model of human capital accumulation throughout formal schooling, following the seminal contribution of Glomm and Ravikumar (1992). The analysis focused on the crucial roles of human capital accumulation in the form of education expenditure. First, we showed that allowance for other accumulated factors makes it possible to model growth as an endogenous process, even when capital physical capital accumulation encounters decreasing returns to scale. The education can be considered as a relevant mechanism in order to make growth an endogenous process in the lung run. Second, education is perhaps more strongly influenced by capital market imperfections than other form of investment, hence it is particular relevant for the distributional perspective. Finally, in light of realistic market imperfections, explicit modeling of education gives an important role to policy intervention and to political interaction. Public education provides a uniform level of schooling which removes the credit constraints that limit the private options of the less affluent; thus it reduces inequality and increases intergenerational mobility. This element can explain why in the contemporary societies the education of the young is overwhelmingly a public responsibility.

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